MODEL OF THE MAIN IONOSPHERIC TROUGH POSITION IN ECCENTRIC DIPOLE COORDINATES

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Based on probe measurement data of electron concentration in the ionosphere from the CHAMP satellite from July 2000 to December 2007, an analysis was conducted on the possibility of using eccentric dipole (ED) coordinates in a model for the invariant latitude of the main ionospheric trough minimum, Φ_m. It was established that the model Φ_m, constructed from these data in corrected geomagnetic (CGM) latitude coordinates, can be used without changes in ED coordinates, since the standard deviation of the model is less than the difference in values of Φ_m for these two variants of geomagnetic latitude specification. The difference in values of Φ_m for these two variants is minimal for the Southern Hemisphere and can be noticeable for the Northern Hemisphere, especially at the longitudes of the East Siberian magnetic anomaly. The dependence of Φ_m on local time and geomagnetic activity is primary. The dependence of Φ_m on geographic longitude is relatively weak, therefore the difference in values of Φ_m between CGM and ED coordinates even at the longitudes of the East Siberian magnetic anomaly is less than the standard deviation of the model.

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1. INTRODUCTION

The International Geomagnetic Reference Field (IGRF) is based on the potential expansion Ψ of the geomagnetic field **V** = – ($\Delta\Psi$) into a series of spherical harmonic terms in spherical geographic coordinates r , θ , λ (see, for example, [Akasofu and Chapman, 1974]). It is assumed that the Earth's surface is spherical with a radius a = 6371.2 km. Taking into account only the first term in the expansion Ψ corresponds to the approximation of **V by the field** of a point axial-centered dipole, whose magnetic moment **M** is located at the center of the Earth and is antiparallel to the Earth's rotation axis [Akasofu and Chapman, 1974]. Accounting for the first three coefficients in the expansion Ψ corresponds to a more accurate approximation of **V by the field** of a point tilted-centered dipole, whose magnetic moment is located at the center of the Earth and is inclined relative to the Earth's rotation axis [Akasofu and Chapman, 1974]. The coordinates of the tilted-centered dipole (TD-coordinates) are often called geomagnetic coordinates. Taking into account the first eight non-zero coefficients in the expansion Ψ allows obtaining an even more accurate approximation of **V by the field** of a point eccentric dipole, whose magnetic moment is equal in magnitude to the moment of the tilted-centered dipole, but is located not at the center of the Earth, but at a point with coordinates r_0, θ_0, λ_0[Akasofu and Chapman, 1974]. The coordinates of the eccentric dipole (ED-coordinates) are obtained from TD-coordinates by parallel transfer of the origin to the point r_0, θ_0, λ_0. Formulas for converting geographic coordinates of a given point to ED-coordinates and vice versa are provided in the works [Fraser-Smith, 1987; Deminov and Fishchuk, 2000].

Approximation **by the field** of a point eccentric dipole allows to study processes along the entire selected geomagnetic field line, and at high altitudes such approximation is quite accurate. Nevertheless, this approximation does not account for regional features of the geomagnetic field, such as the Brazilian and East Siberian magnetic anomalies. A simple and fairly effective method of indirectly accounting for these features is implemented in the corrected geomagnetic (CGM) coordinate system – in geomagnetic coordinates, which are calculated using higher terms of the geomagnetic field expansion in spherical harmonics according to the IGRF model along with dipole terms [Gustafsson et al., 1992]. CGM coordinates are convenient for comparing processes at almost fixed height in the ionosphere with processes at the top of the geomagnetic field line, for example, auroral phenomena. They are inconvenient when comparing environmental parameters at different heights. One way to overcome this disadvantage of CGM coordinates is to introduce altitude-adjusted corrected geomagnetic coordinates (AACGM coordinates), for example, in the height range of 100 - 2000 km [Shepherd, 2014]. Even in this case, for the analysis of processes above 2000 km, tracing along realistic geomagnetic field lines is proposed [Shepherd, 2014].

Therefore, the choice of approximation for the geomagnetic field significantly depends on the type of problems being solved. For the main ionospheric trough (MIT) at altitudes of 300 – 500 km, CGM coordinates (and almost analogous invariant coordinates) are optimal, which is why the majority of models for the MIT minimum position are presented in these coordinates (see references in [Deminov and Shubin, 2018]). When analyzing processes along the entire geomagnetic field line corresponding to the MIT, ED coordinates may be more optimal. It is also possible that a model of the MIT minimum position built in CGM coordinates for an altitude of 400 km could be used in ED coordinates for the same altitude without significantly reducing the model's accuracy. Verifying this assumption was the main goal of this work. The results of solving this problem are presented below. First, the basic equations of the MIT position model in CGM coordinates [Deminov and Shubin, 2018] are presented. They were obtained from probe measurements of electron concentration on the CHAMP (CHAllenging Minisatellite Payload) satellite at altitudes of ~ 350 – 450 km for the years 2000 – 2007. Further, the same data and similar equations were used to obtain a model of the MIT minimum in ED coordinates. Comparing the accuracy of the model equations in CGM and ED coordinates was the basis for solving the stated problem. The implications of this comparison are presented in the Discussion section and summarized in the Conclusions of this work.

2. MINIMUM MIT MODEL IN CGM COORDINATES

This study uses a model of the minimum MIT in CGM coordinates, which is based on electron concentration probe measurements from the CHAMP satellite from July 2000 to December 2007 at altitudes of 400 ± 50 km [Deminov and Shubin, 2018]. This model is a regression equation for the absolute value of the corrected geomagnetic latitude of the MIT minimum, Φ_m, which is called the invariant latitude of this minimum:

$$\Phi_m = 65.5 - 2.4\, Kp^* + \Phi(t) + \Phi(\lambda) \exp(-0.3\, Kp^*), \quad (1)$$

where latitudes Φ_m, $\Phi(t)$, $\Phi(\lambda)$ and geographic longitude λ are defined in degrees, time t is the local magnetic time (MLT) in hours, $\Phi(\lambda) = \Phi_N(\lambda)$ and $\Phi(\lambda) = \Phi_S(\lambda)$ for the Northern and Southern hemispheres respectively,

$$\Phi(t) = 3.16 - 5.6 \cos(15(t - 2.4)) + 1.4 \cos(15(2t - 0.8)), \quad (2)$$

$$\Phi_N(\lambda) = 0.85 \cos(\lambda + 63) - 0.52 \cos(2\lambda + 5), \quad (3)$$

$$\Phi_S(\lambda) = 1.5 \cos(\lambda - 119). \quad (4)$$

In equation (1), the geomagnetic activity index $Kp^* = Kp(\tau)$ for $\tau = 0.6$, where

$$Kp(\tau) = 2.1 \ln(0.2\, ap(\tau) + 1), \quad (5)$$

$$ap(\tau) = (1 - \tau)(ap_0 + ap_{-1}\tau + ap_{-2}\tau^2 + \ldots), \quad (6)$$

ap_0, ap_{-1} and so on are the values of the ap -index for the current, previous, etc. three-hour intervals.

Note that the constant term in $\Phi(t)$ is selected so that at midnight (at $t = 0$), the condition $\Phi(t) = 0$ is satisfied. The index Kp * takes into account the dependence of Φ_m on the history of geomagnetic activity changes.

Model (1) is constructed for a height of 400 km and formally has no limitation on local time and the value of the effective geomagnetic activity index Kp *. However, the original array of SAT minima contained mainly data in the interval 16 - 08 MLT. Therefore, the model is rather qualitative outside this interval, reflecting the general tendency of SAT observations predominantly at night. For the same reason, the array of SAT minima practically did not contain data for local summer. The array of SAT minima mainly contained data for which the condition Kp * < 6 was met. Therefore, the accuracy of the model for Kp * > 6 may not be high.

In model (1), the dependencies of Φ_m on Kp * and local time t are the main ones. Moreover, in many cases in this model, local magnetic time can be replaced by local solar time, since such a replacement leads to an increase in the root-mean-square error of the model by no more than 5 - 10% [Deminov and Shubin, 2018]. In model (1), the dependence of Φ_m on geographic longitude λ is relatively weak. This is taken into account when comparing the accuracy of the SAT minimum model in CGM and ED coordinates.

When constructing model (1), the corrected geomagnetic coordinates of the point were calculated from the known geographic coordinates of this point at an altitude of 400 km using the international geomagnetic field model IGRF-2010, according to the model provided on the Internet (https://omniweb.gsfc.nasa.gov/vitmo). The eccentric dipole coordinates of this point were also calculated using the IGRF-2010 model according to the equations for converting from geographic coordinates to ED coordinates given in the paper [Deminov and Fishchuk, 2000].

3. COMPARISON OF THE ACCURACY OF THE MINIMUM MET MODEL IN CGM AND ED COORDINATES

Let us assume that model (1) can be used without changes in ED coordinates, where the invariant latitude Φ is the modulus of the eccentric dipole latitude at a geographic altitude of 400 km. To evaluate the relative accuracy of model (1) in CGM and ED coordinates, we use the data array of the minimum MET positions in geographic coordinates, on which model (1) in CGM coordinates was based [Deminov and Shubin, 2018]. For greater clarity, we will evaluate the accuracy of the main elements of model (1) in CGM and ED coordinates.

One of these elements of model (1) is the function Φ (*t*), which reflects the dependence of the invariant latitude of the MET minimum on local magnetic time (see equation (2)). It is obtained by determining the coefficients of the regression equation for the invariant latitude of the MET minimum

$\Phi_m(t) = a(t) - b(t)\,Kp^* \pm \sigma(t)$

for each hour *t* = MLT (data in the interval of ± 1 hour relative to a given hour was used), where σ (*t*) is the standard deviation of this equation. For the analyzed interval of 2000 - 2007, the median *Kp* * = 2. In model (1), the coefficient for *Kp* * is constant: *b* (*t*) = 2.4. Therefore, the desired function Φ (*t*) was determined based on the analytical approximation of discrete values

$\Phi^*(t) = (a(t) - 65.5) - (b(t) - 2.4)2 \pm \sigma(t),$

Fig. 1.

which served as experimental data for Φ (*t*). The result is shown in Fig. 1 for the Northern and Southern hemispheres in CGM and ED coordinates. The data in this figure indicates that the difference between experimental data Φ (*t*) in CGM and ED coordinates is maximum in the Northern hemisphere during evening hours (*t* = –8), reaching almost 1.5 ° , but even during these hours, this difference is less than the standard deviation of experimental data in CGM coordinates (2.5 °) and ED (3.1 °). The function Φ (*t*) is constructed using experimental data from the Northern hemisphere in CGM coordinates. In model (1), it is assumed that the function Φ (*t*) can be used without changes for the Southern hemisphere as well, since for this hemisphere the difference between experimental data in CGM coordinates and the function Φ (*t*) is less than the standard deviation of experimental data. Nevertheless, the function Φ (*t*) gives overestimated values of the invariant latitude of the GIP minimum in CGM coordinates in the Southern hemisphere in the local time interval of 0 – 6 hours. For the Southern hemisphere, the difference between experimental data in CGM and ED coordinates is practically non-existent. Therefore, the above-mentioned features of the function Φ (*t*) for the Southern hemisphere are identical in CGM and ED coordinates. Thus, the function Φ (*t*) can be used for both Southern and Northern hemispheres in CGM and ED coordinates, since the differences between Φ (*t*) and experimental data do not exceed the standard deviations of these data.

Model (1) can be represented as

$\Delta\Phi_m = -2.4\,Kp^*, \qquad (7)$

where

$\Delta\Phi_m = \Phi_m - \Phi_m^*,$

$\Phi_{m^*} = 65.5 + \Phi(t) + \Phi(\lambda)\exp(-0.3\,Kp^*)$,

functions $\Phi(t)$ and $\Phi(\lambda)$ are defined by equations (2) – (4). In this case, the calculations of $\Delta\Phi_m$ using experimental data Φ_m and known Φ_{m^*} allow us to assess the accuracy of equation (7), which reflects deviations of Φ_m from the background due to changes in the geomagnetic activity index Kp^* in model (1). The result is shown in Fig. 2 for data in the evening and midnight hours (18 ÷ 03 MLT), when the GIP is most pronounced. This figure shows linear approximations of the experimental data. The standard deviations of these approximations are 2.0 ° and 2.8 ° in CGM and ED coordinates for the Northern Hemisphere and 2.7 ° and 3.0 ° in CGM and ED coordinates for the Southern Hemisphere. Additionally, for these approximations, the proportionality coefficient b is approximately equal to –2.4 ° for the Northern Hemisphere in CGM and ED coordinates and equals –2.72 ° and –2.85 ° for the Southern Hemisphere in CGM and ED coordinates. From the data in Fig. 2, it follows that the linear dependencies of $\Delta\Phi_m$ on Kp^* in CGM and ED coordinates practically coincide in each hemisphere. In the Southern Hemisphere, they differ from equation (7) in model (1). This difference is maximal for $Kp^* = 0$ and is approximately 2 °, which is less than the standard deviation of the linear approximations of the experimental data. This can be seen from the dependence of the function $\Phi(t)$ on local time for $Kp^* = 0$ in Fig. 1. For $Kp^* > 3$, the difference between the linear dependencies of $\Delta\Phi_m$ on Kp^* in CGM or ED coordinates and equation (7) in model (1) is less than 2 ° latitude. Consequently, the linear dependencies of $\Delta\Phi_m$ on Kp^* in CGM and ED coordinates not only practically coincide in each hemisphere but also differ slightly between hemispheres, except during periods of very low geomagnetic activity. This means that equation (7) in model (1), constructed in CGM coordinates, is also valid for ED coordinates. For the Southern Hemisphere, the standard deviations of equation (7) in CGM and ED coordinates practically coincide. For the Northern Hemisphere, these deviations in CGM coordinates are about 0.8 ° latitude less than in ED coordinates.

Fig. 2.

In model (1), the longitudinal features of the MID minimum position are taken into account using the function $\Phi(\lambda)$. From the equations of model (1), it can be seen that the longitudinal effect is maximal at low geomagnetic activity ($Kp^* = 0$) and even in this case, this effect usually does not exceed the standard deviation of the model. The function $\Phi(\lambda)$ depends in a certain way on the longitudinal variations of the geographic latitude φ at a fixed invariant latitude $\Phi = 65$ °, which is denoted as $|\varphi(65, \lambda)|$ [Deminov and Shubin, 2018]:

$$\Phi(\lambda) = -0.13 (|\varphi(65, \lambda)| - 65). \qquad (8)$$

Fig. 3.

Formula (8) is applicable for the Northern and Southern hemispheres, therefore the absolute value $\varphi(65, \lambda)$ is used in this formula. These dependencies in CGM and ED coordinates are shown in Fig. 3.

From the data in this figure, it can be seen that in CGM coordinates, the dependence of $\Phi(\lambda)$ on λ according to equation (8) leads to an error compared to equations (3) and (4) of model (1) less than 1 ° of latitude. This is also true for the dependence of $\Phi(\lambda)$ on λ according to equation (8) in ED coordinates. For the Southern hemisphere, there is no difference between the values of $\Phi(\lambda)$ according to equation (8) in CGM and ED coordinates. For the Northern hemisphere, it is more noticeable, but less than 1 ° of latitude.

Thus, in general, model (1), constructed in CGM coordinates for an altitude of 400 km, can be used without changes in ED coordinates for this altitude, since the difference is less than the standard deviation of the model. For the Southern hemisphere, there is practically no difference in the values of the MID minimum position in CGM and ED coordinates. For the Northern hemisphere, this difference is more noticeable and, moreover, for this hemisphere, the variance in the values of the MID minimum position in CGM coordinates is less than in ED coordinates.

4. DISCUSSION

The results presented above showed that the model of the main ionospheric trough (MIT) minimum position, created in CGM coordinates, can be used in ED coordinates without modifications, since the difference between these two approaches does not exceed the standard deviation of the model. The difference between CGM and ED coordinates at an altitude of 400 km is mainly due to large-scale regional features of the geomagnetic field that are not accounted for in the ED model. In the Southern hemisphere, such a regional feature is the Brazilian anomaly, which is located at relatively low latitudes where the MIT is not typically observed. Therefore, in the Southern hemisphere, there is practically no difference in the positions of the MIT minimum in CGM and ED coordinates. The East Siberian magnetic anomaly in the Northern hemisphere covers a vast area and is located at typical MIT latitudes. Therefore, in the Northern hemisphere, the difference in the positions of the MIT minimum in CGM and ED coordinates is more noticeable than in the Southern hemisphere (see Fig. 3). Nevertheless, this difference does not exceed the standard deviation of the model.

The properties of model (1) and its comparison with some other models of the MIT minimum position, Φ_m, are presented in the article [Deminov and Shubin, 2018]. Here we will note only some features of model (1). This model is relatively simple and is intended primarily for practical application. For example, the dependence of Φ_m on *Kp* * is assumed to be linear, independent of local time, and the same for the Northern and Southern hemispheres. Nevertheless, model (1) takes into account the dependence of Φ_m on the history of geomagnetic activity changes through the index *Kp* *. Usually, the condition $Kp^* < Kp$ is satisfied, since *Kp* * is a weighted average index (see equations (5) and (6)). Therefore, other conditions being equal, the proportionality coefficient for *Kp* * is greater than for *Kp* , (see, for example, [Annakuliev et al., 1997]). This should be taken into account when comparing models of the MIT minimum position [Aa et al., 2020; Karpachev, 2024].

Satellite data on which the results of the presented analysis are based mainly refer to night hours during equinoxes and local winter, when the MITs were recorded quite distinctly. Therefore, data arrays for the position of MIT in the Northern and Southern hemispheres do not coincide. For example, to obtain the dependence Φ_m on *Kp* *, 5426 and 3845 measurements of Φ_m in the Northern and Southern hemispheres, respectively, were used. The difference in the dependencies of Φ_m on *Kp* * for the Northern and Southern hemispheres, which is noticeable during low geomagnetic activity, may be related to this feature of the data arrays.

5. CONCLUSIONS

Based on data from probe measurements of electron concentration in the ionosphere on the CHAMP satellite from July 2000 to December 2007, an analysis was conducted on the possibility of using eccentric dipole (ED) coordinates in the model for the invariant latitude of the main ionospheric trough minimum, Φ_m. The following conclusions were obtained.

1. The model of Φ_m, built on these data in corrected geomagnetic (CGM) latitude coordinates, can be used without changes in ED coordinates, since the standard deviation of the model is less than the difference in values of Φ_m for these two variants of geomagnetic latitude specification.
2. The difference in values of Φ_m for these two variants is minimal for the Southern Hemisphere and may be noticeable for the Northern Hemisphere, especially at the longitudes of the East Siberian magnetic anomaly.

3. The dependence of Φ_m on local time and geomagnetic activity is fundamental. The dependence of Φ_m on geographic longitude is relatively weak, therefore the difference in values of Φ_m between CGM and ED coordinates even at the longitudes of the East Siberian magnetic anomaly is less than the standard deviation of the model.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

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Figure Captions

Fig. 1. Diurnal variations of the function Φ (*t*) according to equation (2) (solid line) and experimental data in CGM coordinates (dots) and ED (diamonds). Vertical lines show standard deviations of experimental data in CGM coordinates.

Fig. 2. Deviations of the ionospheric trough minimum latitude from the background $\Delta\Phi_m$ due to changes in the geomagnetic activity index *Kp* * according to equation (7) (thick lines) and experimental data in CGM coordinates (thin lines) and ED (dashed lines).

Fig. 3. Dependence of the function Φ (λ) on geographic longitude λ according to equations (3) and (4) (thick lines) and equation (8) in CGM and ED coordinates (thin and dashed lines, respectively).

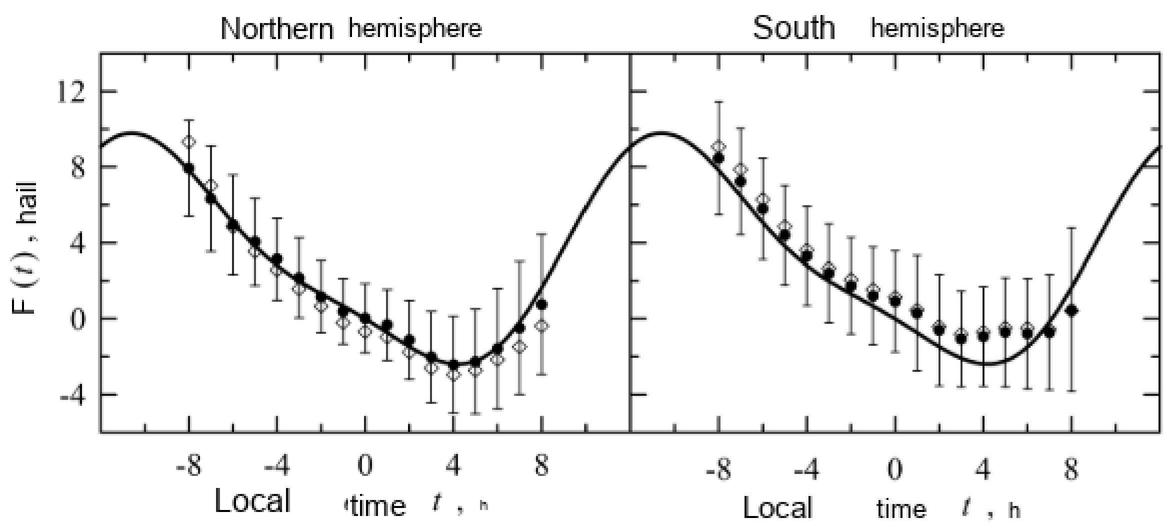
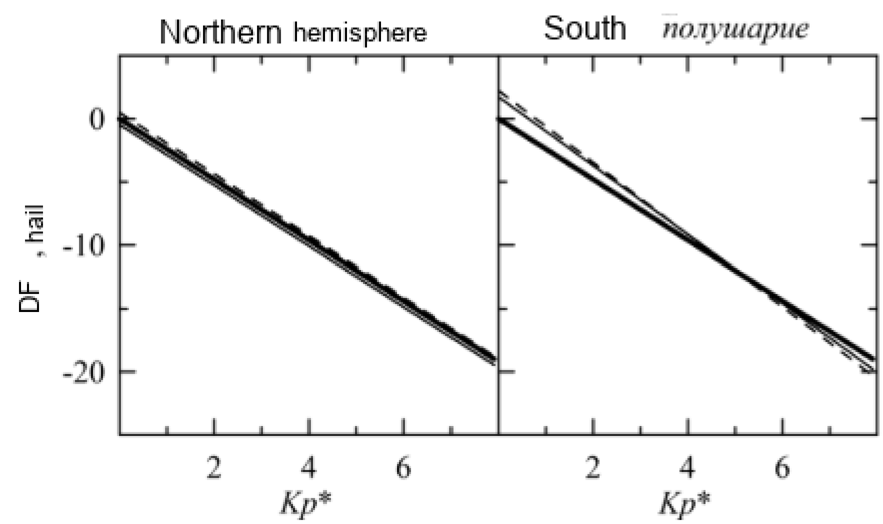
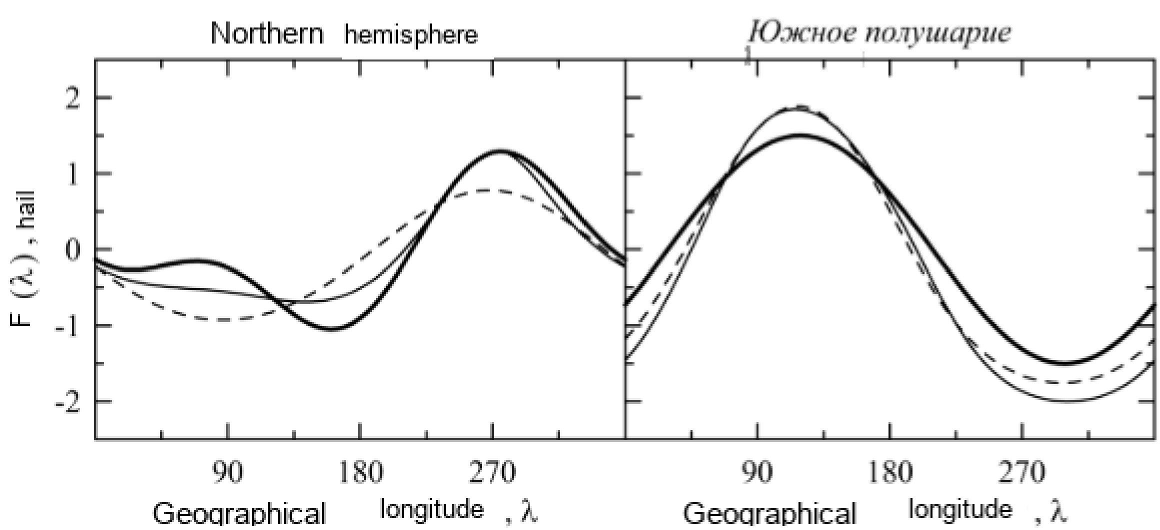


Fig. 1.



Northern hemisphere
South полушарие
0
-10
-20
DF , hail
2
4
6
Kp*
2
4
6
Kp*

Fig. 2.



Northern hemisphere
Южное полушарие
2
1
0
-1
-2
F (λ) , hail
90
180
270
Geographical longitude , λ
90
180
270
Geographical longitude , λ

Fig. 3.