ULTRAHIGH 11-YEAR CYCLES BASED ON RADIOCARBON RECONSTRUCTIONS OF SOLAR ACTIVITY

© 2025 D. M. Volobuev[a, *], I. V. Kudryavtsev[a,b], N. G. Makarenko[a]

[a]*Main (Pulkovo) Astronomical Observatory of the Russian Academy of Sciences, St. Petersburg, Russia*

[b]*Ioffe Institute of Physics and Technology, Russian Academy of Sciences, St. Petersburg, Russia*

**e-mail: dmitry.volobuev@mail.ru*

Received February 19, 2025

Revised April 20, 2025

Accepted June 17, 2025

Abstract. The curve of radiocarbon dating with decade time step has been known for more than half a century and is the main, most accurate way of chronology of archaeological finds. Overlapping patterns of ring thicknesses of preserved tree remains allow to build a chronology for more than 10000 years. A number of authors have already obtained reconstructions of secular variations of solar activity for the Holocene period. The accuracy of modern mass spectrometers allows us to work with a small amount of material, so there are more and more chronological curves with a one-year time step, lasting up to 1000 and more years. Such curves, theoretically, should reflect the main, 11-year variations of solar activity. However, the amplitude of 11-year variations in radiocarbon content is comparable to the measurement error, and the trajectory of a radiocarbon atom from its formation in the atmosphere to its arrival in the tree ring as a result of carbon exchange between natural reservoirs turns out to be very confusing and subject to a number of changing factors. In this paper, we discuss possible approaches to reconstructing 11-year solar activity cycles from radiocarbon and ways to improve their accuracy. In particular, we derive a number of high (up to 500 Wolf units) 11-year cycles at the end of the 12th century that may not be apparent when over-smoothing is applied.

DOI: 10.31857/S00167940250711e3

1. INTRODUCTION

The relative radiocarbon content $\Delta^{14}C$ in tree rings is a product of the interaction of galactic cosmic rays (GCR) with atmospheric nitrogen and reflects changes in solar activity (SA), due to modulation of cosmic rays by the interplanetary magnetic field [Damon et al., 1973], which varies with changes in SA. Time series of weathering radiocarbon content are of great interest to solar physics and can be used to investigate the past behavior of 11-year cycles [Damon et al., 1973]. In this case, to estimate the number of sunspots from the relative radiocarbon content of tree rings in general is necessary:

1) Solve the inverse problem of carbon fluxes from the atmosphere to the ocean, soil, biosphere, etc., by calculating the rate of radiocarbon production in the atmosphere.

2) Take into account the rigidity of the cutoff of the GCR spectrum by the Earth's magnetic field by calculating the heliospheric modulation potential and the so-called "open" magnetic flux.

3) Recalculate the "open" magnetic flux into the number of sunspots.

Each of these steps is beset by both additional data measurement errors and modeling errors. One recent radiocarbon series [Brehm et al., 2021] was nevertheless converted to sunspot number [Usoskin et al., 2021], numbering all 11-year cycles over 1000 years, and indicating the amplitudes of their maxima as well as the epochs of their minima. Given the uncertainties associated with the above steps, we find the reliability of these results questionable. In addition, the transformation proposed by the authors for the third step uses some controversial assumptions, in particular, the correction of the "slow" variable and a quadratic transformation that translates the so-called "open" magnetic flux into the number of sunspots [Usoskin et al., 2021], and, consequently, reduces the modulation of the cycle minima.

To resolve these doubts, in this paper we use our empirical [Volobuev and Makarenko, 2015] model strengthened by Ensemble Empirical Mode Decomposition (EEMD) and the 5-reservoir model [Dorman, 1978] modernized to account for climatic changes [Kudryavtsev et al., 2016; Kudryavtsev et al., 2018; Larionova et al., 2020], which we refer to here as "neoclassical". Using these two models, we attempt to assess possible model errors and the reliability of radiocarbon reconstructions of 11-year cycles in tree rings.

The aim of this study is not only to refine the solar activity reconstructions but, more to the point, to identify potential limitations and sources of errors in existing methods.

2. DATA

We used year-to-year data on the relative radiocarbon content of tree rings obtained in the USA [Stuiver et al., 1998] and data obtained in the UK and Switzerland using a state-of-the-art mass spectrometer [Brehm et al., 2021]. Figure 1 compares these data on an overlapping interval - it can be seen that the differences in these regional data on a time scale of 11 years can exceed the amplitude of the 11-year cycle. For the reconstruction on the overlapping interval, we used a composite series obtained as the average of these two series weighted by their inverse errors σ:

.

In Figure 1, this series - $\Delta^{14}C$ inverted in sign. The series is spline smoothed to ensure its differentiability in functional form, with the smoothing parameter chosen so that the 11-year variation is visible in the raw data (Figure 1).

The Northern Hemisphere temperature series reconstructed from tree ring thicknesses [Crowley, 2000] and the atmospheric carbon dioxide series reconstructed from Antarctic ice boreholes [Etheridge et al., 1998] were also used for the "neoclassical" reconstruction. In addition, data on past changes in the Earth's dipole magnetic moment from archaeomagnetic data were used [Genevey et al., 2008].

3. EEMD RECONSTRUCTION

The reconstruction is based on finding the coefficients for the inverse diffusion problem using the least squares method [Volobuev and Makarenko, 2015] for the time interval 1700-1950, based on the composite series Δ^{14}C smoothed by splines (Fig. 1).

In this case, taking advantage of the analogy between the heat conduction and diffusion equation, we used the formalism developed for solving the inverse problem of heat conduction (Beck et al., 1985) considering the problem of the change in the concentration of carbon *C* in the "ocean":

$$\frac{\partial}{\partial x}\left(k\frac{\partial C}{\partial x}\right) = \lambda\frac{\partial C}{\partial t}, \quad (1)$$

where we need to find the variation of the unknown flux *q* at the boundary ($x = 0$), between the "atmosphere" and the "ocean":

$$q(t) = -k\frac{\partial C(x,t)}{\partial x}. \quad (2)$$

For a semi-infinite medium, the Duhamel integral solution is known, which for discrete (stepwise) changes in flux and concentration at time t_i takes the following form:

$$q_E(t_M) = 2\sqrt{\frac{k\lambda}{\pi}}\sum_{i=1}^{M}\frac{C_i - C_{i-1}}{\sqrt{t_M - t_i} + \sqrt{t_M - t_{i-1}}}, \quad (3)$$

We consider this solution here as the first iteration for Burggraf's solution [Burggraf, 1964] in the case of a zero-thickness layer, *E 0,* $= q_E = q_M$. *M* = *L - 1*, where *L* is the row length.

$$q(t) = q_E(t) + k\sum_{n=1}^{3}\frac{E^{2n-1}}{(2n-1)!}\frac{1}{a^n}\frac{d^n C}{dt^n} + \sum_{n=1}^{3}\frac{E^{2n}}{(2n)!}\frac{1}{a^n}\frac{d^n q_E}{dt^n} \quad . \quad (4)$$

The summation over *n* here is up to 3, which guarantees the differentiability of the cubic spline. *E-thickness of* the layer. It should be noted here that the formulation of the problem (1-4) is extremely simplified and is not quite rigorous, since it leaves an additional adjustment coefficient λ, which has no direct physical meaning for the diffusion equation, but gives the model additional flexibility, which is insufficient if one selects only the coefficients *a*, *K*, and *E*. In contrast to [Volobuev and Makarenko, 2015], the inverse problem (1-4) was solved for separate empirical modes obtained using small noise additives, the so-called EEMDs [Wu and Huang, 2009]. Since the frequency of the empirical modes decreases with the mode number, we conditionally associate each mode with an increasingly deeper diffusion layer that increasingly smoothes the original multi-frequency variation q(t) , but the coefficients chosen are purely empirical and do not make physical sense. The EEMD decomposition used 10 realizations with 1% noise added. The model was calibrated using the SSN sunspot number series (Fig. 4) available at WDC-SILSO, Royal Observatory of Belgium, Brussels. The result of the decomposition in the form of individual modes is shown in Fig. 3. The trend in EEMD, as in conventional EMD, is the last numbered mode, and is calculated as the residual of the decomposition, under the condition that the sum of all modes is equal to the original series, all modes except the trend are zero mean (symmetric about zero) by construction. Then, for each of the modes, the coefficients of the inverse problem (1-4) were selected by the Levenberg-Marquardt method, and $q(t)$ calculated for individual modes (images of the modes) were summarized to obtain the final result. The fitted coefficients for the 3 empirical modes and the trend took the following values:

a = [0.1956 ; 0.0903; 0.0435; 0.0256], λ = [0.1138; 0 ; 0.1268; 0],

k = [43.931; 10.773; 39.434; 45.315], E = [1.0655; 0.6762; 0.0537; 0.0154].

Thus, using 4 free parameters, we constructed an empirical model to reconstruct sunspot numbers from radiocarbon in tree rings over the interval of the last millennium. The result is compared with the others in Fig. 4-6.

4. "NEOCLASSICAL" RECONSTRUCTION

The reconstruction is based on the papers [Kudryavtsev et al., 2016; 2018; Larionova et al., 2020], but uses the $\Delta^{14}C$ composite series as input (Fig. 1). In contrast to the paper [Usoskin et al., 2021], the "neoclassical" reconstruction uses a 5-reservoir model, atmospheric CO_2 content data, and global temperature change data. The change in atmospheric CO_2 content must be accounted for because $\Delta^{14}C$ describes the deviation from the standard value of the ratio of the ^{14}C isotope concentration to the ^{12}C isotope concentration. Therefore, $\Delta^{14}C$ varies with both changes in the concentration of isotope (^{14}C and CO_2 concentration (see also [Roth and Joos, 2013]). Changes in temperature affect the rate of carbon exchange between natural reservoirs, in particular between the ocean and the atmosphere (which is accounted for in the paper) as a result of the temperature dependence of carbon dioxide solubility in water. Based on these data, we reconstruct the formation rate of the ^{14}C isotope in the Earth's atmosphere and the Heliospheric Modulation Potential. In doing so, our reconstruction, similar to [Brehm et al., 2021] and [Larionova et al., 2020], takes into account the change in the stiffness of the GCR cutoff as the Earth's past magnetic dipole moment changes from archaeomagnetic data. To distinguish 11-year cycles in the "neoclassical" reconstruction, trend removal is performed after calculating the heliospheric modulation potential and, similar to [Usoskin et al., 2021], a quadratic transformation is applied to reduce the modulation at minima and translate the modulation potential to sunspot number scale. However, in contrast to

[Usoskin et al., 2021], the "neoclassical" reconstruction does not use the decadal radiocarbon series to correct the long-term envelope, and smoothing of the fast peaks in the original $\Delta^{14}C$ series. Therefore, several high 11-year cycles in the late 12th century and early 14th century can be observed in this reconstruction (Figures 4-6). The shift in the positions of the extrema of the 11-year cycles relative to the other reconstructions is due to the inclusion of long-term changes in CO_2 and temperature, as well as the lack of pre-smoothing and trend subtraction. This indicates additional phase uncertainty introduced by these factors.

5. RESULTS AND CONCLUSIONS

The existing measurement errors (Fig. 2) and model errors, different but not zero in different models, do not reliably reconstruct 11-year cycles from radiocarbon data. The model errors are related to the integrating effect of the basin equations, which simulate primarily the dissolution of carbon dioxide in the ocean. Accordingly, the inverse problem involves the need for numerical differentiation, which multiplies the measurement error proportional to frequency. Excessive smoothing with a wide window, while reducing noise, can significantly reduce the amplitude of short high cycles. In particular, the Savitzky-Golay filter used by the authors of [Brehm et al., 2021] and [Usoskin et al., 2021] may not be optimal in this sense. On the other hand, taking into account low-frequency factors such as ocean surface temperature, CO_2variations, or the Earth's dipole magnetic moment, whose data have large error, the smoothing sequence and its parameters may introduce uncertainty in the position of the cycle extrema on the time axis (Fig. 5).

In general, as shown by the EEMD model, high 11-year cycles are recovered more reliably than low ones because of the larger signal-to-noise ratio for the time derivatives computed from the data when solving the inverse diffusion problem or the basin equations.

Based on the available data, we hypothesize the presence of unprecedented high 11-year cycles in the late 12th century, above the highest 19th cycle, according to Zurich numbering, that occurred in the 20th century. An indirect consequence of this may have been the well-known medieval warming of the climate, leading in particular to the establishment of Viking settlements on the Greenland coast.

ACKNOWLEDGEMENTS

We thank two anonymous reviewers for careful reading of the article and helpful comments. We also thank the authors supporting open databases, without which this work would not have been possible.

FUNDING

This work was funded by the budgets of the authors' organizations. No additional grants were received to conduct or direct this particular study.

CONFLICT OF INTERESTS

The authors declare that they have no conflicts of interest.

REFERENCES

1. *Dorman L.I.,* 1978. Features of the study of cosmic ray variations by the radiocarbon method. Proceedings of the sixth All-Union conference on the problem of "Astrophysical phenomena and radio carbon". Tbilisi, October 13-15, 1976. Tbilisi University Press. Tbilisi 1978. P.49-96.
2. *Brehm N., Bayliss A., Christl M., et al.,* 2021. Eleven-year solar cycles over the last millennium revealed by radiocarbon in tree rings // Nature Geoscience, 14(1), pp.10-15
3. *Crowley T.J.,* 2000. Causes of climate change over the past 1000 years // Science, 289(5477), pp.270-277.
4. *Beck J.V., Blackwell B., and Clair C.R.St.,* Inverse Heat Conduction, New York: Wiley, 1985.
5. *Burggraf O.R.,* 1964. An Exact Solution of the Inverse Problem in Heat Conduction Theory and Applications // J. Heat Transfer., v. 86(3), pp.373-380.
6. *Damon P.E., Long A. and Wallick E.I.,* 1973. On the magnitude of the 11-year radiocarbon cycle // Earth and planetary science letters, 20(3), pp.300-306.
7. *Etheridge D.M., Steele L.P., Langenfelds R.L., et al.,* 1998. Historical CO2 record derived from a spline fit (75 year cutoff) of the Law Dome DSS, DE08, and DE08-2 ice cores.
8. *Genevey A., Gallet Y., Constable C.G., et al.,* 2008. ArcheoInt: An upgraded compilation of geomagnetic field intensity data for the past ten millennia and its application to the recovery of the past dipole moment // Geochemistry, Geophysics, Geosystems, 9(4).
9. *Kudryavtsev I.V., Volobuev D.M., Dergachev V.A., et al.,* 2016. Reconstructions of the ^{14}C cosmogenic isotope content from natural archives after the last glacial termination // Geomagnetism and Aeronomy, 56, pp.858-862.
10. *Kudryavtsev I.V., Volobuev D.M., Dergachev V.A., et al.,* 2018. Reconstruction of the Production Rate of Cosmogenic ^{14}C in the Earth's Atmosphere for 17 000–5000 BC // Geomagnetism and Aeronomy, 58, pp.925-929.
11. *Larionova A.I., Dergachev V.A., Kudryavtsev I.V., et al.,* 2020. Radiocarbon data from the late 18th century as a reflection of solar activity variation, natural climate change, and anthropogenic activity // Geomagnetism and Aeronomy, 60, pp.840-845.
12. *Roth R. and Joos F.,* 2013. A reconstruction of radiocarbon production and total solar irradiance from the Holocene ^{14}C and CO_2 records: implications of data and model uncertainties // Clim. Past, 9, pp. 1879–1909, 2013

13. *Stuiver M., P. J. Reimer and T. F. Braziunas.* High-Precision Radiocarbon Age Calibration for Terrestrial and Marine Samples // Radiocarbon 40, 1127-1151 (1998)
14. *Usoskin I.G., Solanki S.K., Krivova N.A., et al.,* 2021. Solar cyclic activity over the last millennium reconstructed from annual ^{14}C data // Astronomy & Astrophysics, 649, p.A141.
15. *Volobuev D.M. and Makarenko, N.G.,* 2015. Radiocarbon version of 11-year variations in the interplanetary magnetic field since 1250 // Geomagnetism and Aeronomy, 55, pp.938-944.
16. WDC-SILSO, Royal Observatory of Belgium, Brussels https://www.sidc.be/SILSO/datafiles
17. *Wu Z. and Huang N.E.,* 2009. Ensemble empirical mode decomposition: a noise-assisted data analysis method // Advances in adaptive data analysis, 1(01), pp.1-41.

FIGURE CAPTIONS

Fig. 1. Original weathering series Δ^{14}C on an overlapping time interval (1400-1500), and the composite inverted series used for modeling, Δ^{14}C smoothed with a spline so that 11-year variations with a maximum amplitude of about 2.5 ppm relative to the long-term trend can be seen.

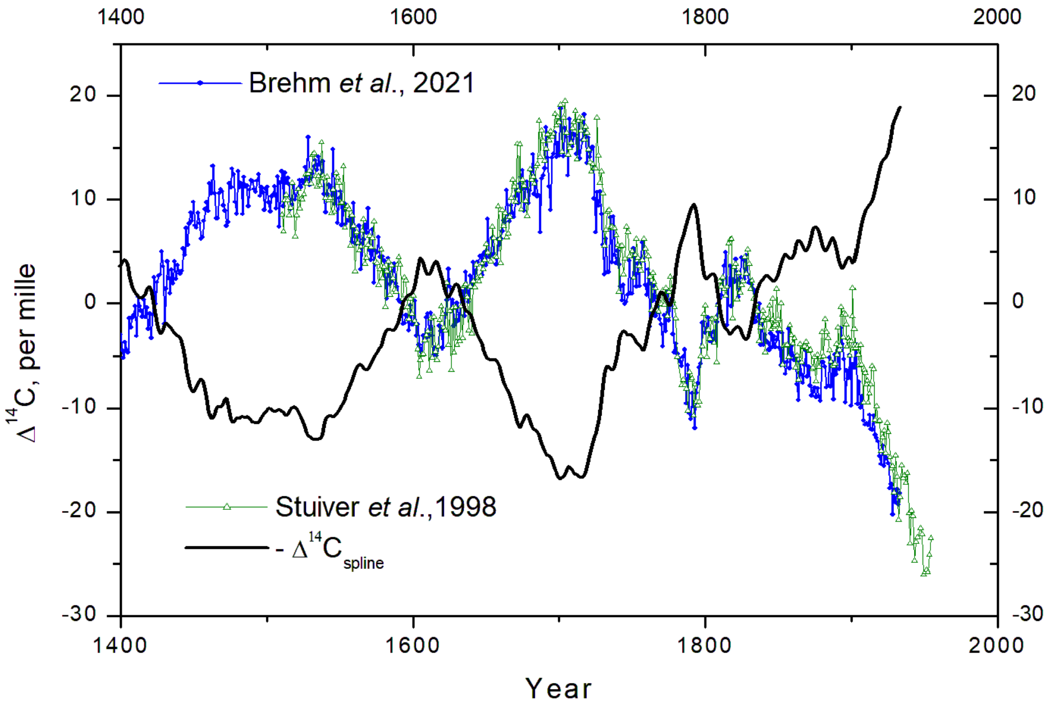
Fig. 2. A section of the series [Brehm et al., 2021] at the end of the 12th century showing a large amplitude, about four ppm, for the rapid Δ^{14}C variations on the 11-year scale compared to the average amplitude of the 11-year variation in radiocarbon characteristic of the present (see Fig. 1). The measurement errors are large, so that cycles of small amplitude are not formally detectable, and the amplitude of large cycles will be determined with a large error, since the model error is added to the measurement error.

Fig. 3. Empirical modes of the EEMD decomposition.

Fig. 4. Comparison of model solutions with a range of sunspot numbers (SSN) on the "calibration" interval.

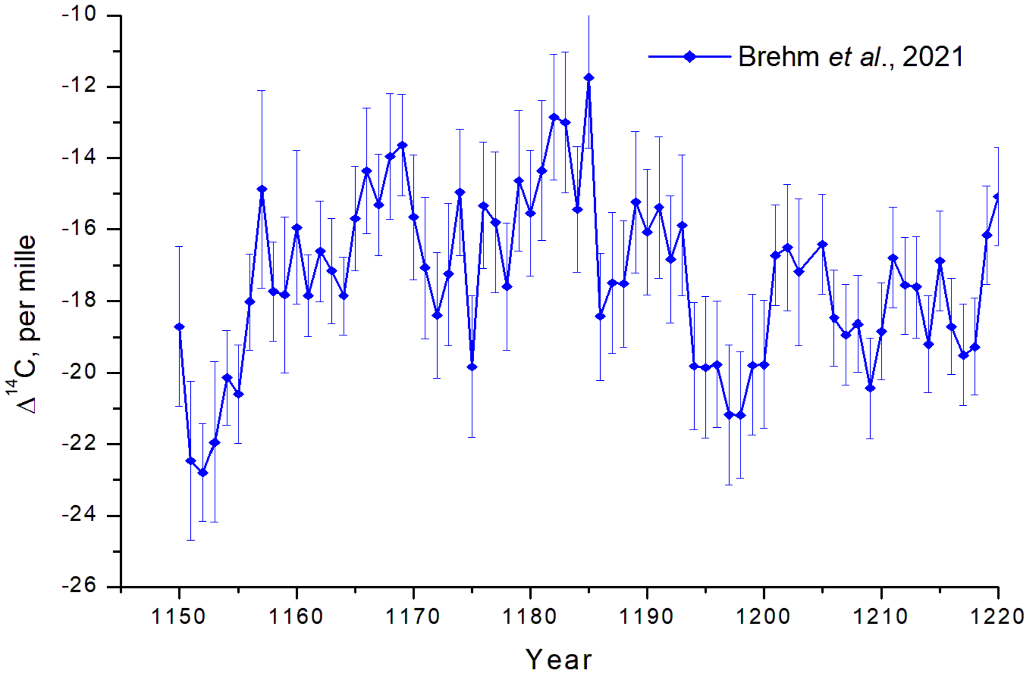
Fig. 5. Reconstructions of solar activity from 1000 years of weathering radiocarbon.

Fig. 6. High cycles of the late 12th century.



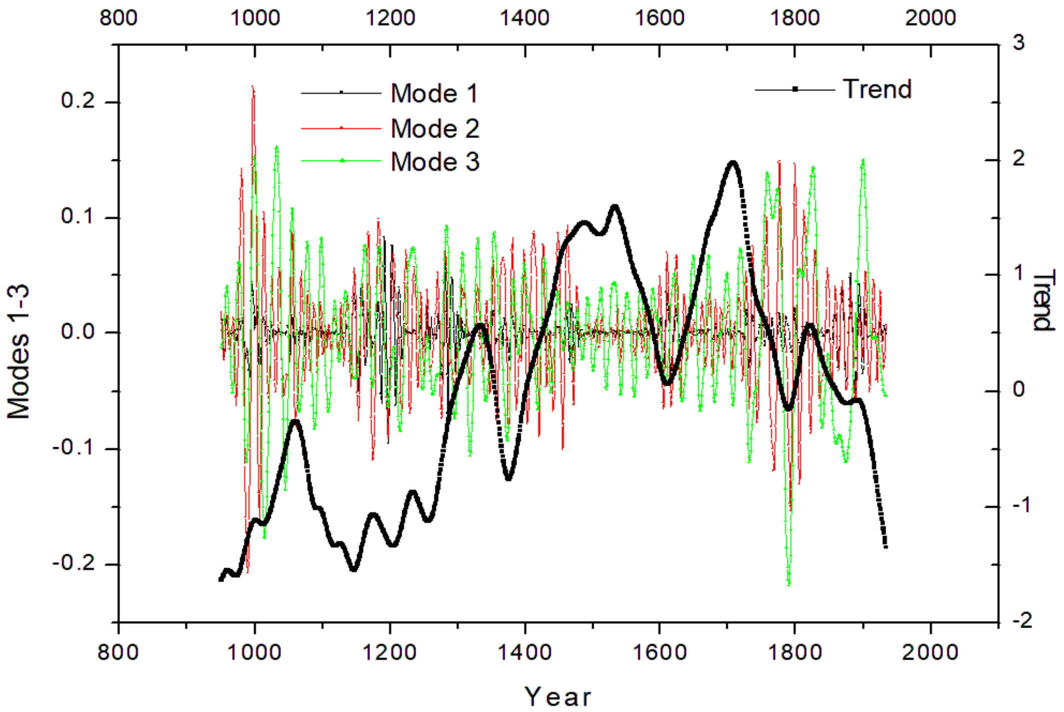
Brehm et al., 2021
Stuiver et al.,1998
- Δ14C spline
Δ14C, per mille
Year

Fig. 1.



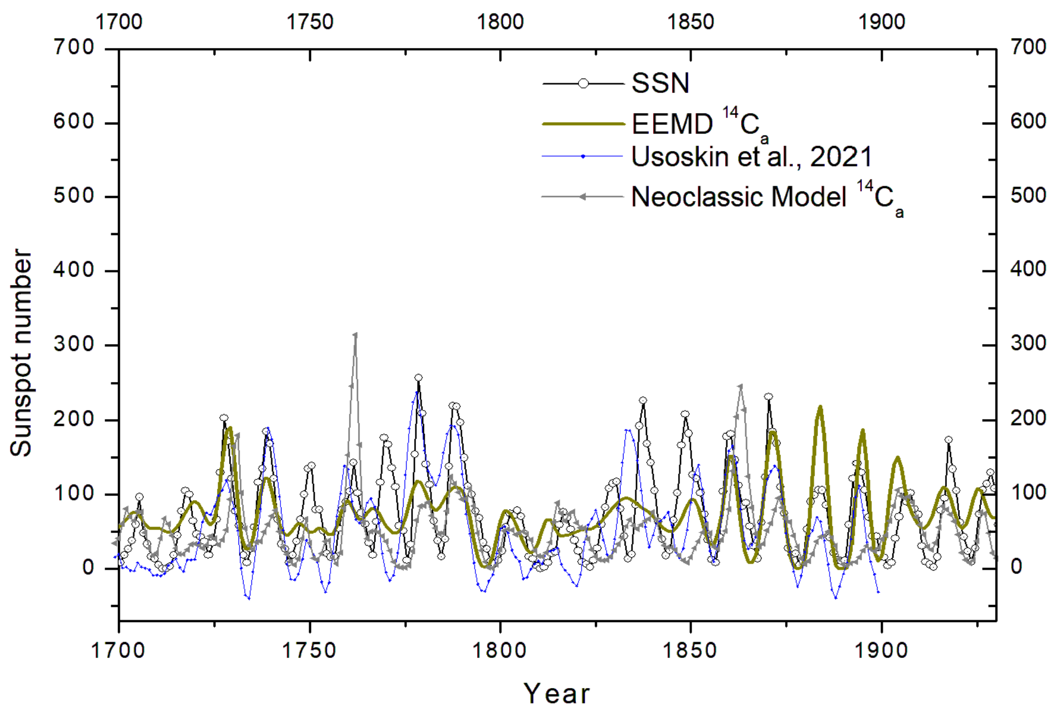
Brehm et al., 2021
Δ14C, per mille
Year

Fig. 2.



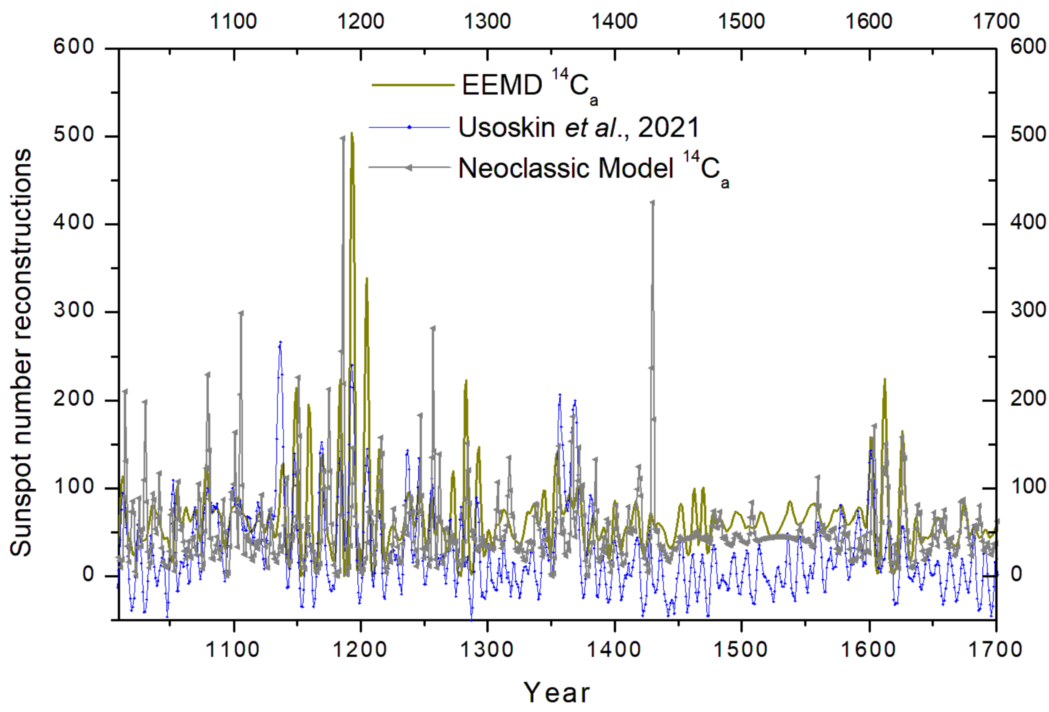
Mode 1
Mode 2
Mode 3
Trend
Modes 1-3
Trend
Year

Fig. 3.



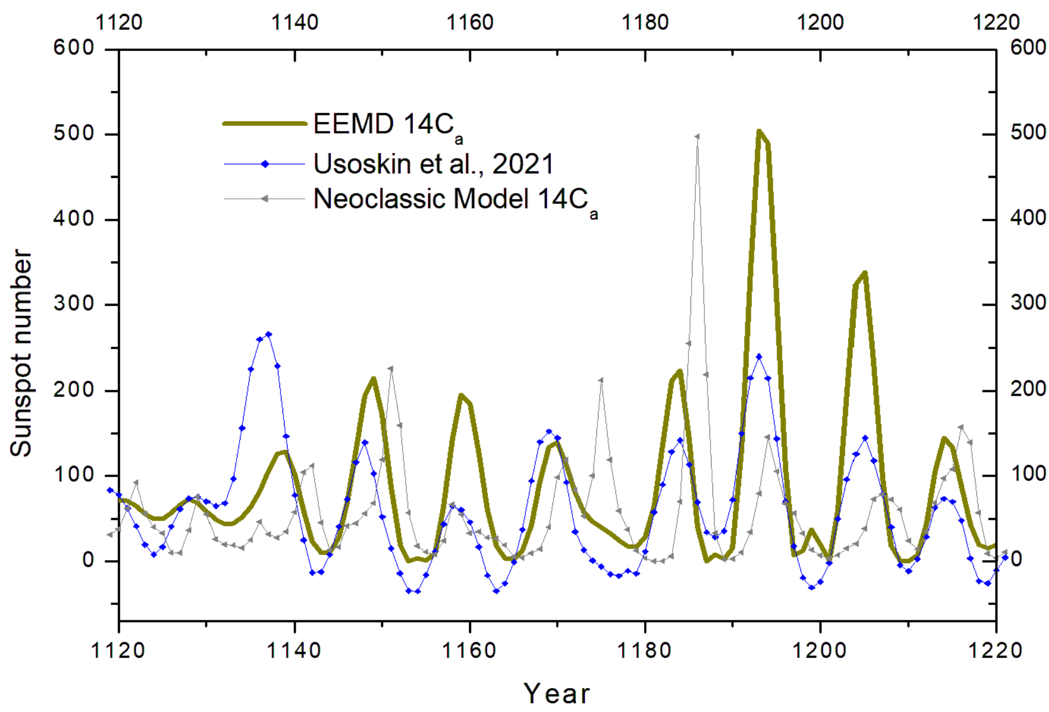
SSN
EEMD ¹⁴Cₐ
Usoskin et al., 2021
Neoclassic Model ¹⁴Cₐ
Sunspot number
Year

Fig. 4.



EEMD $^{14}C_a$
Usoskin *et al.*, 2021
Neoclassic Model $^{14}C_a$
Sunspot number reconstructions
Year

Fig. 5.



EEMD $14C_a$
Usoskin et al., 2021
Neoclassic Model $14C_a$
Sunspot number
Year

Fig. 6.