MODELING OF TIME DEPENDENCIES
OF THE MEAN VALUES OF THE VELOCITY AND MAGNETIC FIELD
IN THE CONVECTIVE ZONE OF THE STAR

© 2025 R. A. Kislov*, S. V. Starchenko**

Pushkov Institute of Terrestrial Magnetism, Russian Academy of Sciences, Moscow, Troitsk, Russia.

**e-mail: kr-rk@bk.ru*

***e-mail: sstarchenko@mail.ru*

Received February 03, 2025

Revised April 23, 2025

Accepted June 17, 2025

Abstract. Qualitative estimates concerning the dynamics of the convective zone of a star (CZ) as a whole are useful both in the conditions of lack of detailed observational information about the star and as a preliminary step before building a more complex model requiring laborious calculations. In this paper, we present a qualitative model describing the evolution of the mean values of the velocity and magnetic field squares in the convective zone of a Sun-like star. The stability of possible equilibrium values of the mean squares of velocity and magnetic field is investigated, and solutions of the model equations at different values of buoyancy and the ratio of the matter and magnetic field convection times are obtained. It is shown that there are possible scenarios in which 1) the magnetic field strengthens, having any small initial value; 2) the magnetic field disappears, being initially finite; 3) the behavior of velocity and magnetic field near stationary values and far from them may differ significantly. The amplification/weakening of the RMS magnetic field does not depend on the initial conditions and is determined only by the parameters of the CMZ. The parameters of the convective zone of the Sun correspond to the boundary case between 1 and 2 and their small changes can lead to different scenarios.

Keywords: *convective zone of a star, Sun, stellar magnetic fields, convection*

DOI: 10.31857/S00167940250719e2

1. INTRODUCTION

The convective zone of a star (CZ) is the region inside a star in which the conditions for the occurrence of thermal convection are fulfilled. The CPZ in red dwarfs can occupy most of the volume between the core and the photosphere, in sun-like stars the CPZ is located between the radiative transfer zone and the photosphere, and can be 20-30% of the stellar radius. In stars of spectral class A the EPC has the form of a narrow layer near of the photosphere, and in stars of spectral classes O, B the EPC is located around the core. In red dwarfs, the EPC can occupy most of the volume [Pickellner, 1986].

Among all CPZs, the convective zone of the Sun is the best studied. The available data are based on helioseismological data and the results of modeling the structure of the Sun [Guenther et al., 1992]. The convective zone of the Sun is characterized by significant gradients of matter density, which varies within the zone by 7 orders of magnitude, from $2.5 \cdot 10^{-4}$kg/m^3 near the photosphere to $6.6 \cdot 10^3$kg/m^3 at the base of the zone. The temperature varies in the solar CPZ by 3 orders of magnitude, from $5.8 \cdot 10^3$ K at the photospheric surface to $5.8 \cdot 10^6$K at the base of the convective zone [Götling, 2022; Guenther et al., 1992]. The peak diffusion velocity in the solar CPZ is less than 25 m/s [Chizaru et al., 2010]. The degree of ionization of helium and hydrogen near the photosphere is much higher than at the base of the KZZ. The latter reduces the temperature difference in the KZZ, since the energy entering each pop-up plasma element goes mainly to ionization rather than heating [Pickelner, 1986]. These circumstances complicate analytical and numerical modeling of the KZZ, making this problem not yet completely solved [Götling, 2022; Elliott and Smolarkiewicz, 2002].

It is believed that the processes in the KZZ play a key role in the formation of the magnetic field and activity cycle of the Sun and similar stars [Guenther et al. 1992; Getling 2001; Chizaru et al., 2010; Getling, Kosovichev, 2022]. Modern models take into account the equations of momentum transfer, energy transfer, the influence of chemical composition and ionization on the equation of state of matter, radiative transfer and energy release in the core, as well as viscous and magnetic diffusion, stellar rotation, and plasma compressibility [Guenther et al., 1992; Elliott and Smolarkiewicz, 2002; Zasov and Postnov, 2006; Chizaru et al., 2010].

The complexity of modern models is so great that the reproduction of the results obtained and their interpretation in different conditions are comparable in labor intensity to the creation of a new model, which was noted in [Elliott, Smolarkiewicz, 2002]. Another problem is the limited data on the magnetic field, velocity, and plasma density of other stars. The stellar disk is often indistinguishable and one has to be satisfied with only disk-averaged quantities. In view of the above, it is natural to use simplified estimation models, including those that consider the SGC as a whole and operate with mean magnitudes or mean squares of magnitudes [Kislov and Starchenko, 2024].

Such an approach is common in problems related to the geomagnetic dynamo [Braginsky, Roberts, 1995; Glatzmaier, Roberts, 1995; Cristensen et al., 2010; Starchenko, 2019] and is successfully used along with models that consider in detail the structure of the outer core and mantle [Shebalin, 2025].

The purpose of this work is to evaluate the character of the evolution of the mean squares of the magnetic field and velocity in the KZZ at different ratios of the viscous and magnetic diffusion times and at different buoyancy. Also, a study of the stability of the equilibrium values of the mean squares of the magnetic field and velocity in the KZZ at constant values of the parameters is performed.

2. THE SYSTEM OF EQUATIONS, DESCRIBING THE MEAN SQUARE VALUES OF VELOCITY AND MAGNETIC FIELD IN THE CONVECTIVE ZONE OF THE STAR

Let us represent the convective zone of a star (CZZ) as a part of a sphere enclosed between radii r_0 and r_1. Inside the CZZ there is a non-zero magnetic field $\boldsymbol{B}$ and plasma convection velocity $\boldsymbol{v}$, related by the Navier-Stokes equation including the Ampere force and gravity densities, and written in a frame of reference rotating with the Carrington cyclic frequency ω:

$$\rho\frac{\partial \boldsymbol{v}}{\partial t}+\rho(\boldsymbol{v},\nabla)\boldsymbol{v}=-\nabla P+\frac{1}{4\pi}[rot\boldsymbol{B},\boldsymbol{B}]+\rho\boldsymbol{g}+\eta\Delta\boldsymbol{v}+2\rho[\boldsymbol{v},\boldsymbol{\omega}]+\rho[[\boldsymbol{\omega},\boldsymbol{r}],\boldsymbol{\omega}], \tag{1}$$

where ρ is the plasma density, P is the thermal pressure, $\boldsymbol{g}$ is the vector of free fall acceleration, η is the dynamic viscosity, the last two summands are related to the densities of the forces of inertia - Coriolis and centrifugal. The current density is excluded using Ampere's equation. Let's take into account the finite conductivity of the medium. Then there will be a violation of the magnetic field embeddedness in the plasma due to the presence of magnetic viscosity:

$$\frac{\partial \boldsymbol{B}}{\partial t}=rot[\boldsymbol{v},\boldsymbol{B}]+\eta_m\Delta\boldsymbol{B}\ , \tag{2}$$

where η_m is the magnetic viscosity. Since the model will be of an evaluative nature, the viscosities can be considered as constants to simplify the calculations. We will also consider the continuity equation:

$$\frac{\partial\rho}{\partial t}+div(\rho\boldsymbol{v})=0 \tag{3}$$

Let us introduce $\boldsymbol{A}$ - the acceleration of surfacing caused by the deviation of thermal pressure from hydrostatic pressure and directed along the velocity $\boldsymbol{v}$, $\rho\boldsymbol{A}=\frac{(\boldsymbol{v},-\nabla P+\rho\boldsymbol{g}+\rho[[\boldsymbol{\omega},\boldsymbol{r}],\boldsymbol{\omega}])}{v^2}\boldsymbol{v}$. The vector $\boldsymbol{A}$ ***is*** independent of Coriolis acceleration by virtue of its definition, but may include a correction to gravity associated with centrifugal acceleration. Multiply scalarly equation (1) by the velocity, which eliminates the Coriolis force and conditions the definition of $\boldsymbol{A}$, and integrate over the volume of the convective zone. We obtain from (1) and (3) under the assumption that convection vanishes near the boundaries of the zone and the summands having the form of an integral of the divergence $\int_{r_0}^{r}\sum_i \partial_i(f_i \cdot scalar)dO$ can be excluded:

$$\frac{d}{dt}\int\frac{\rho v^2}{2}dO = \int\left(\rho(\boldsymbol{A},\boldsymbol{v}) - \frac{([\boldsymbol{v},\boldsymbol{B}], rot\boldsymbol{B})}{4\pi} - \eta(rot(\boldsymbol{v}))^2 - \eta(div(\boldsymbol{v}))^2\right)dO, \quad (4)$$

where dO is the volume element and d/dt denotes the convective derivative $\frac{\partial}{\partial t} + (\boldsymbol{v},\nabla)$. Equation (4) differs from that used in [Starchenko, 2019, 2024] by the presence of velocity divergence associated with compressibility, which cannot be neglected in the CPZ [Guenther et al., 1992; Chizaru et al., 2010; Götling, 2022].

Let us multiply scalarly equation (2) by the magnetic field. Acting in the same way as in the derivation of (4), we obtain

$$\frac{d}{dt}\int\frac{B^2}{8\pi}dO = \int\left(\frac{([\boldsymbol{v},\boldsymbol{B}], rot\boldsymbol{B})}{4\pi} - \eta_m(rot(\boldsymbol{B}))^2\right)dO, \quad (5)$$

where the left part of (5) contains the convective derivative similarly to (4). In the derivation, the divergence-free magnetic field was taken into account, **divB = 0**.

Note that the summand associated with the presence of the Ampere force contains the velocity and is quadratic in the magnetic field. When averaging over the volume, one could assume that this summand could go to zero. However, the other summands in (4, 5) are either sign-constant or, on average, not equal to zero (as the product of surfacing acceleration and velocity), so the average value of the Ampere force density is different from zero. Equations (4, 5) describe the evolution of the kinetic and magnetic energies of the KZZ.

We introduce the notations ρ_1, V and H for the volume-averaged KZZ density and the squares of the velocity and magnetic field. We proceed to order-of-magnitude estimates by replacing all derivatives in space by division by the spatial scale L of the velocity and magnetic field changes in the KZZ. In this case, we limit ourselves to the case when these scales are equal. We also introduce buoyancy $a_1 = (\boldsymbol{A}, \boldsymbol{v})$. Thus we obtain a system of equations, allowing us to make estimates of the time dependence of the RMS values of the velocity and magnetic field:

$$
\begin{cases} H\dfrac{dH}{dt} = \dfrac{VH^2}{L} - 4\pi\eta_m\dfrac{H^2}{L^2} \\ V\dfrac{dV}{dt} = a_1 - \dfrac{VH^2}{4\pi L\rho_1} - \eta\dfrac{V^2}{\rho_1 L^2} \end{cases} \tag{6}
$$

Let us introduce dimensionless variables as follows. Let $H = B_0 b$, $V = v_0 u$, $t = T\tau$, $a_1 = a_0\tau/v_0^2\cdot\alpha$, where quantities with index 0 are units of magnetic field, velocity and buoyancy, and T is time. Moreover, we will choose the units so that $L = v_0 T$. Then equations (6) will contain three parameters - dimensionless buoyancy α, matter convection time $T_u = \rho_1 L^2/\eta$ and magnetic field convection time $T_b = L^2/4\pi\eta_m$. For convenience, we can choose the units so that $\tau_u = \tau$, so finally we have a system with two parameters where $\tau_b = T_b/T_u$.

$$
b\frac{db}{d\tau} = ub^2 - \frac{b^2}{\tau_b} \tag{7}
$$

and

$$
u\frac{du}{d\tau} = \alpha - u^2 - ub^2 \tag{8}
$$

The system of equations (7, 8), despite the simplifications, can only be solved numerically. In the following, we will work with equations (7, 8). Note that in this paper and in the papers [Starchenko, 2019, 2024] the system (7, 8) has the same form, but they are obtained under different assumptions, buoyancy in (8) and convective time in (7) have a different definition. For convective time, the difference is that it takes into account the effect of compressibility. Buoyancy, on the other hand, is of a different nature because the Boussinesq approximation is not applicable in the CPZ, and thermal pressure cannot be discarded when writing (4) explicitly because the density is not a constant.

3- INVESTIGATION OF THE EQUILIBRIUM POSITIONS OF THE SYSTEM

The system of equations (7, 8) has two stationary points with positive u, b.

The first stationary point: ($u = \alpha^{1/2}$, $b = 0$), exists at $\alpha \geq 0$.

The second stationary point: ($u = 1/\tau_b$, $b = (\alpha\tau_b - 1/\tau_b)^{1/2}$), exists at $\alpha\tau_b^2 \geq 1$.

Previously, the stability of these points was investigated in [Starchenko, 2024], but the analysis of the types of stationary points and the behavior of the solutions of the system (7, 8) at different parameters was not carried out.

3.1 Study of the first stationary point, ($u = \alpha^{1/2}$, $b = 0$)

Consider small deviations from the equilibrium position $u = \alpha^{1/2} + \delta u$, $b = \delta b$. The system (7, 8) after linearization will take the form:

$$\begin{cases} \dfrac{d\delta b}{dt} = \left(\sqrt{\alpha} - \dfrac{1}{\tau_b}\right)\delta b \\ \dfrac{d\delta u}{dt} = -2\delta u \end{cases} \tag{9}$$

The characteristic equation of system (9) has roots $\alpha^{1/2} - 1/\tau$ and -2. If $\alpha\tau_b^2 > 1$, the stationary point is of saddle type, in the opposite case it is a stable node. When $\alpha^{1/2} - 1/\tau + 2 = 0$ or when $\alpha\tau_b^2 = 1$, the equilibrium position is non-coarse, i.e., the nature of the solutions of the system (7, 8) is not determined by the behavior near this stationary point [Romanko, 2001]. The final conclusion about the kind of solutions of the system in any case can be made only after a formal analysis of the second stationary point. Note that the linearized system is easy to solve, but the image of its solutions is not of interest in itself.

3.2 Investigation of the second stationary point, ($u = 1/\tau_b$, $b = (\alpha\tau_b - 1/\tau_b)^{1/2}$)

Let $u = 1/\tau_b + \delta x$, $b^2 = \alpha\tau_b - 1/\tau_b + \delta y$. Then the system (7, 8) takes the following form after linearization:

$$\begin{cases} \dfrac{d\delta y}{dt} = 2\left(\alpha\tau_b - \dfrac{1}{\tau_b}\right)\delta x \\ \dfrac{d\delta x}{dt} = -(\alpha\tau_b^2 + 1)\delta x - \delta y \end{cases} \tag{10}$$

The characteristic equation of the system (10) has solutions

$$z = \frac{1}{2}\left(-\alpha\tau_b^2 - 1 \pm \sqrt{(\alpha\tau_b^2 + 1)^2 - 8\left(\alpha\tau_b - \frac{1}{\tau_b}\right)}\right) \tag{11}$$

The subscript expression in (11) is the discriminant, which we denote as D. For system (10), the following cases are possible:

1. $D < 0$. Then the stationary point has the type of a stable focus, since the real part of (11) is negative.

2. $D > 0$. Since the second stationary point exists at $\alpha\tau_b^2 \geq 1$, $z \leq 0$. The cases $z < 0$ correspond to the type of stable knot, $z = 0$ (possibly at $\alpha\tau_b^2 = 1$) corresponds to a non-coarse

equilibrium position.

3. If $D = 0$, then the equilibrium is non-coarse.

3.3 Joint analysis of stationary points

If $\alpha\tau_b^2 >$ 1, then the first stationary point is unstable, but depending on the sign of *D,* the second stationary point will be of the type of either a stable focus or a stable node. Then the solutions of the system (7, 8) will tend to equilibrium ($u = 1/\tau_b$, $b = (\alpha\tau_b - 1/\tau_b)^{1/2}$) regardless of the initial conditions. This means that no matter how small the initial magnetic field is, it will increase to a macroscopic value, which can be considered as a dynamo effect. In the case of a stable focus, the real part of (11) is less than -1, so the oscillations of the parameters will decay very quickly and the solution curve (7, 8) will not have time to make more than one revolution when approaching equilibrium. Therefore, this case will not describe anything resembling an activity cycle.

If $\alpha\tau_b^2$1, then only the first stationary point exists, and it will be of the type of a stable node. In such a case, whatever the initial magnetic field is, it will tend to zero.

The cases $\alpha\tau_b^2 = 1$, $\alpha^{1/2} - 1/\tau_b + 2 = 0$ (when τ_b is small and there is no second stationary point) or *D=0* remain boundary cases and correspond to non-coarse equilibria. Analytical study of such cases is difficult.

Note that in the approximation under study there is no question of the minimum magnetic field in the CPZ, at which its growth to macroscopic values is possible. This is a question of parameter values rather than the choice of initial conditions.

4. EXAMPLES OF SOLUTIONS

Let us consider variants of parameter values corresponding to the six cases described above - three coarse equilibria and three non-coarse equilibria. The solutions in all cases are obtained by the fourth-order Runge-Kutta method. The parameters in all cases are chosen to demonstrate typical solutions.

1) Let $\tau_b = 1$, $\alpha = 4$. Then the stationary points are (2, 0) and ($1, 3^{1/2}$), the discriminant in (11) is positive, $\alpha\tau_b^2 > 1$. The former is of saddle type and unstable, while the latter is a stable node. Fig. 1 shows the solutions for the dimensionless (a) rms velocity u, (b) RMS magnetic field b, (c) parametric dependence b(u). The initial conditions are chosen ($u = 0.1$, $b = 0.1$).

Fig. 1.

This case describes the establishment of a non-zero magnetic field. If it was initially less than its equilibrium value, there will be an increasing solution; in the opposite case, there will be a decreasing solution. When the initial values of b are equal or close to the stationary value, the solution will be non-monotonic. Deviations of b from the initial value in this case are related to changes in u.

2) Let $\tau_b = 0.5$, $\alpha = 64$, $\alpha\tau_b^2 > 1$, $D<0$. Then the stationary points are (8, 0) and $(2, (63/2)^{1/2})$. The first point is again of saddle type. The second point has the type of a stable focus with significant damping and determines the type of solutions. The solutions for this case are shown in Figure 2.

Fig. 2.

The sharp initial growth of velocity in Fig. 2a and magnetic field in Fig. 2b is associated with high buoyancy. It causes u, b to exceed the steady-state values and, as a consequence, the integral curve in Fig. 2c to twist near equilibrium. This case describes the establishment of a magnetic field with quasi-oscillations. Full-fledged oscillations are not possible in this problem because $1 + \alpha\tau_b^2 > 1$, which corresponds to a characteristic dimensionless damping time less than 1. We do not consider the cases of negative buoyancy because they are not characteristic of the CPZ and the solutions of the system (7, 8) will be unstable.

3) Let $\tau_b = 0.5$, $\alpha = 1$, and $\alpha\tau_b^2 < 1$. Then the first stationary point is equal to (1/2, 0) and is of the type of a stable node, and the second one is nonexistent. The solutions are shown in Fig. 3.

Fig. 3.

This case describes the disappearance of the magnetic field, however large it may be initially. The velocity u is different from 0 and can grow in the case where the initial value is smaller than the equilibrium value.

4) Let $\tau_b = 2$, $\alpha = 0.25$, which corresponds to $\alpha\tau_b^2 = 1$, a non-coarse equilibrium. In this case both stationary points coincide, (1/2, 0). The behavior of the solutions is unusual and requires consideration of longer time intervals than in the other cases (Fig. 4).

Fig. 4.

As can be seen from Fig. 4b, the magnetic field is unusually slow compared to the previous cases to reach its equilibrium value, while the velocity becomes stationary no slower than before (Fig. 4a). At t = 100, the magnetic field lies between 0.06 and 0.07. Even at t = 1000, the magnetic field is close to 0.02 and remains of the same order as the initial value. In such a regime, the star is sensitive to external influences, for example, due to the close passage of a large planet or a companion star with an elongated orbit. Then the decreasing regime can easily change to a slow growth or, on the contrary, to a faster decrease.

5) Let $\tau = 1/3$, $\alpha = 1$, then $\alpha^{1/2} - 1/\tau_b + 2 = 0$, $\alpha\tau_b^2 < 1$, so that there is only the first stationary point (1, 0), and the equilibrium turns out to be non-coarse. The solutions in this case are similar to those obtained for a stable node. Fig. 5 shows how non-monotonicity arises when the initial value of u coincides with the stationary value.

Fig. 5.

6) Let $\tau = 1$, $\alpha = 3$, then both stationary points $(3^{1/2}, 0)$, $(1, 2^{1/2})$ exist. The first one is unstable since it is of saddle type, the second one turns out to be a non-gross equilibrium since the eigenvalues of (11) coincide, $D = 0$. The solutions for u and b in this case are shown in Fig. 6.

Fig. 6.

In this case, the solutions are similar to case 1, but with a small change in parameters the stationary point can change type to a stable focus, which, however, is only of mathematical interest since noticeable changes in the shape of the curves occur at large buoyancy (of order 10 or more). Importantly, this case does not lead to any unusual and significant effects for physical interpretation, despite the mathematical isolation.

5. DISCUSSION AND CONCLUSIONS

A system of equations describing the mean squares of the velocity and magnetic field in the convective zone of a star (CZ) is obtained and studied. Previously, the approach in which the CPZ was considered as a whole in the context of the magnetic dynamo was not widespread; the main emphasis was placed on detailed models of the CPZ. The following results are obtained:

1. The system of equations (7, 8) describing the dynamics of the RMS values of velocity and magnetic field in the KZZ was investigated.
2. The system of equations (7, 8) can have no more than two stationary states. One of them is always stable.
3. The stationary values do not depend on the initial conditions.
4. Scenarios are possible in which any small mean square of the magnetic field in the KZZ reaches macroscopic values. In this case, the initial magnetic field has no threshold value below which amplification is impossible. Within the framework of this approach, the magnetic dynamo in the short-circuit zone is a matter of the short-circuit zone parameters, not of the initial conditions.
5. Parameter values at which the initially finite magnetic field vanishes are possible.
6. Examples of solutions of the system (7, 8) for the parameters at which the equilibria turn out to be non-coarse are considered. The regime at which the disappearance of the magnetic field is orders of magnitude slower than in the case of coarse equilibria is found.

In the future, it is planned to study solutions at variable parameters, as well as possible modifications of the system (7, 8).

Finally, it is of interest to estimate the parameters $\tau_b$and α for the convective zone of the Sun. Following the transition from (6) to (7, 8), we see that $\tau_b = T_b/T_u = \eta / \rho\eta_m$, in other words, it is the ratio of kinematic and magnetic viscosities. The kinematic viscosity is variously estimated to range from 10^{12}cm^2/s to 10^{14}cm^2/s [Unno, Ribes, 1976; Vandakurov, 1976; Kuznetsov, Syrovatsky, 1979]. The value of $6\cdot10^{12}$cm^2/s corresponds best to the observations [Kuznetsov and Syrovatsky, 1979]. The magnetic viscosity is taken in agreement with [Maiewski et al., 2022], η_m= 10^{13}cm^2/s.

Thus, τ_b= 0.1-10, with a value of 0.6 in agreement with [Kuznetsov and Syrovatsky, 1979]. Estimating the surfacing acceleration A on the order of magnitude as the local free-fall acceleration (~250 m/s$^{2)}$, magnetic fields of the order of 10^2-10^3Gs [Unno, Ribes, 1976; Krivodubsky, 1987], the size of the convective zone L ~ 10^{10}cm [Spruit, 1974; Guenther et al, 1992; Getling, Kosovichev, 2022], the velocity of convective currents of the order of 10^2-10^3cm/s [Vandakurov, 1976; Goldreich, Keeley, 1977; Chizaru et al., 2010; Getling, 2022], we obtain an estimate of α ~ 0.1-1, which, in the presence of buoyancy alone, corresponds to a surfacing time of the order of 10 years. We can assume that buoyancy alone can provide a faster resurfacing time, since the presented version of the model describes only a part of the cycle (away from the minimum or maximum, where the functions in (4, 5) may have time extrema). Then the surfacing time is of the order of one year, and the parameter α can reach 10. Thus, $\alpha\tau_b^2$ is of order 1, so both the cases of magnetic field enhancement and attenuation are possible. If the buoyancy and/or mixing time change with time (which is natural), one mode may succeed the other. The boundary condition of the CPZ may be due to the fact that the Sun has a cycle. In the case $\alpha\tau_b^2 + (\alpha\tau_b^2)^{-1} >> 1$, the cycle will be impossible because the magnetic field will either disappear forever or grow to a certain value and remain unchanged.

FUNDING

This work was funded from the Institute's budget. No additional grants were received to conduct or supervise this particular study.

CONFLICT OF INTERESTS

The authors confirm that they have no conflicts of interest.

REFERENCES

1. *Vandakurov Yu.V.* Solar convection and the 11-year cycle. Leningrad: Nauka Publ., 1976, 156 p.
2. *Getling A.V.,* Convective mechanism of formation of photospheric magnetic fields // Astronomical Bulletin, 2001, V. 78, No. 7, P. 661-668.
3. *Getling A.V.,* Convective zone of the Sun. The Great Russian Encyclopedia, 2022. https://bigenc.ru/c/konvektivnaia-zona-solntsa-261f37
4. *Zasov A.V., Postnov K.A.,* General Astrophysics. Fryazino: Century 2, 2006, 496 p.
5. *Kislov R.A., Starchenko S.V.,* Evolution of the total magnetic and kinetic energy in the convective zone of a star. Proceedings of the XXVIII All-Russian Conference “Solar and Solar-terrestrial Physics", Russia, St. Petersburg, 2024, P. 161-164. https://doi.org/10.31725/0552-5829-2024-161-164
6. *Krivodubsky V.N.,* Letters to the Astronomical Journal, 1987, V. 13, P. 803-810 *(English*

version: https://ui.adsabs.harvard.edu/abs/1987SvAL...13..338K/abstract)

7. *Kuznetsov V.D., Syrovatsky S.I.,* Buoyancy of magnetic fields and the 11-year cycle of solar activity // Astronomical Journal, 1979, vol. 56, pp. 1263-1271. (English version: https://ui.adsabs.harvard.edu/abs/1979SvA....23..715K/abstract)
8. *Pickelner S.B.,* Convection. Physics of space. A small encyclopedia. Edited by R.A. Sunyaev. Moscow: Soviet Encyclopedia, 1986, 786 p.
9. *Romanko V.K.,* Course of differential equations and calculus of Variations. 2nd ed. Moscow: Laboratory of Basic Knowledge, 2001, 344 p.
10. *Braginsky S.I., Roberts P.H.,* Equations governing convection in the Earth's core and the geodynamo // Geoph. Astroph. Fluid Dyn. 1995, vol. 79, pp. 1–97.
11. *Chizaru M., Carbonneau P., Smolarkiewicz P.K.,* Magnetic cycles in global large-eddy simulations of solar convection // Astrophys. J. Lett, 2010, vol. 715, pp. L133-137. doi: 10.1088/2041-8205/715/2/L133
12. *Christensen U., Aubert J., Hulot G.,* Conditions for Earth-like geodynamo models // Earth Planet. Sci. Lett. 2010, vol. 296, pp. 487–496.
13. *Elliott J.R., Smolarkiewicz P.K.,* Eddy resolving simulations of turbulent solar convection // Int. J. Numer. Meth. Fluids, 2002, vol. 39, pp. 855–864. doi: 10.1002/fld.333
14. *Getling A.V., Kosovichev A.G.,* Spatial scales and time variation of solar subsurface convection // Astrophys. J., 2022, vol. 937:41, 12 pp. https://doi.org/10.3847/1538-4357/ac8870
15. *Glatzmaier G.A., Roberts P.H.,* A three-dimensional convective dynamo solution with rotating and finitely conducting inner core and mantle // Phys. Earth Planet. Int. 1995, vol. 91(1–3), pp. 63–75.
16. *Goldreich P., Keeley D.A.,* Solar seismology. II. The stochastic excitation of the solar p-modes by turbulent convection // Astrophys. J., 1977, vol. 212, pp. 243-252.
17. *Guenther D.B., Demarque P., Kim Y.-C., Pinsonneault M.H.,* Standard solar model // Astrophys. J., 1992, vol. 387, pp. 372-393. doi: 10.1086/171090
18. *Maiewski E., Malova H., Popov V., et al.,* Migrating dynamo waves and consequences for stellar current sheets // Solar Physics, 2022, vol. 297:150, 27 pp. https://doi.org/10.1007/s11207-022-02085-3
19. *Shebalin J.V.,* Magnetohyrodynamic turbulence in a spherical shell: Galerkin models, boundary conditions, and the dynamo problem // Fluids, 2025, vol. 10 (2), 24. https://doi.org/10.3390/fluids10020024
20. *Spruit H.C.,* A model of the solar convection zone // Solar Physics, 1974, vol. 34, issue 2, pp. 277-290. doi: 10.1007/BF00153665

21. *Starchenko S.V.,* Analytic scaling laws in planetary dynamo models // Geoph. Astroph. Fluid Dyn., 2019, vol. 113, no 1-2, pp. 71-79. https://doi.org/10.1080/03091929.2018.1551531
22. *Starchenko S.V.,* Simple Model of the Evolution of Magnetic and Kinetic Energy of the Geodynamo // Geomagnetism and Aeronomy, 2024, vol. 64, no. 6, pp. 890–896. doi: 10.1134/S0016793224600759
23. *Unno W., Ribes E.,* On magnetic buoyancy in the convection zone // Astrophys. J., 1976, vol. 208, pp. 222-223. doi: 10.1086/154597

FIGURE CAPTIONS

Fig. 1. Dimensionless solutions of the system (7, 8) when α= 4, τ_b= 1, and at initial conditions (u = 0.1, b = 0.1). (a) - velocity, (b) - magnetic field, (c) integral curves, magnetic field dependence on velocity. Dimensionless time varies from 0 to 10, the number of steps is 50000.

Fig. 2. Dimensionless solutions of the system (7, 8) at α= 64, τ_b= 1/2, and at initial conditions (u = 0.1, b = 0.1). (a) - velocity, (b) - magnetic field, (c) integral curves, dependence of magnetic field on velocity. Dimensionless time varies from 0 to 10, the number of steps is 50000.

Fig. 3. Dimensionless solutions of the system (7, 8) when α= 1, τ_b= 1/2, and at initial conditions (u = 0.1, b = 0.1). (a) - velocity, (b) - magnetic field, (c) integral curves, dependence of magnetic field on velocity. Dimensionless time varies from 0 to 10, the number of steps is 50000.

Fig. 4. Dimensionless solutions of the system (7, 8) at α= 0.25, τ_b= 2, and at initial conditions (u = 0.1, b = 0.1). (a) - velocity, (b) - magnetic field, (c) integral curves, magnetic field dependence on velocity. Dimensionless time varies from 0 to 100, the number of steps is 50000.

Fig. 5. Dimensionless solutions of the system (7, 8) when α= 1, τ_b= 1/3, and at initial conditions (u = 1, b = 1). (a) - velocity, (b) - magnetic field, (c) integral curves, magnetic field dependence on velocity. Dimensionless time varies from 0 to 10, the number of steps is 50000.

Fig. 6. Dimensionless solutions of the system (7, 8) when α= 3, τ_b= 1, and at initial conditions (u = 0.1, b = 0.1). (a) - velocity, (b) - magnetic field, (c) integral curves, magnetic field dependence on velocity. The dimensionless time varies from 0 to 10, the number of steps is 50000.

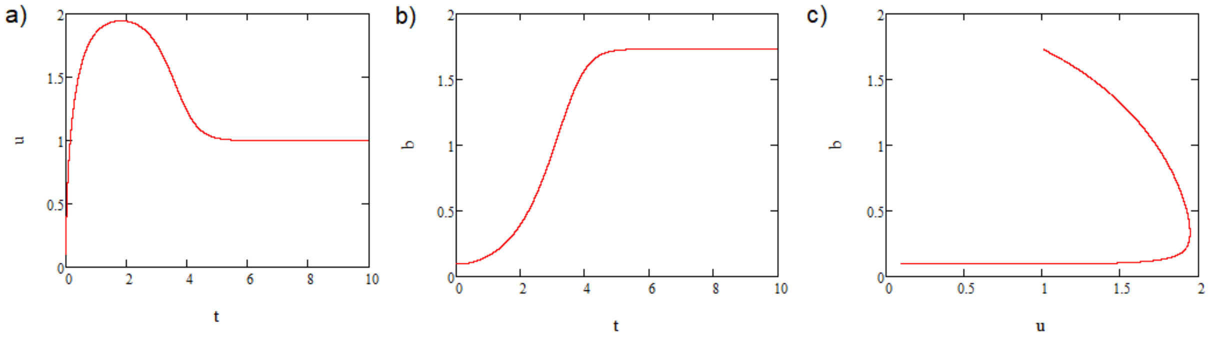

Fig. 1.

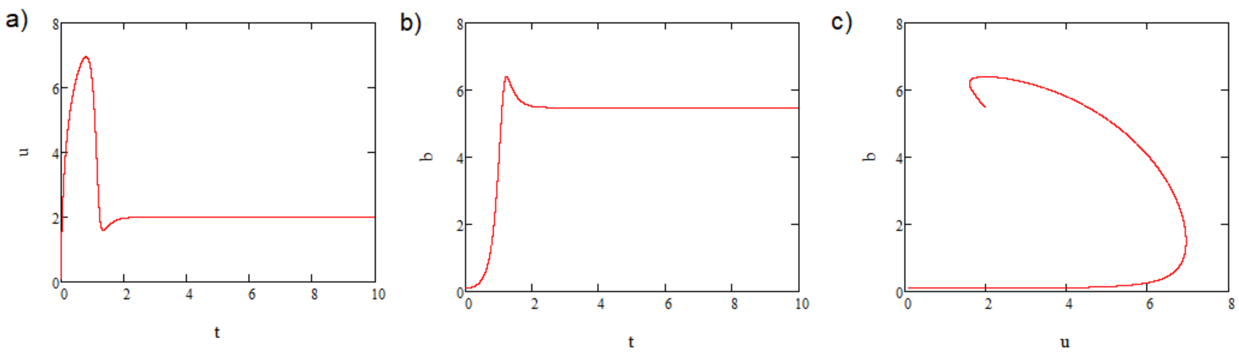

Fig. 2.

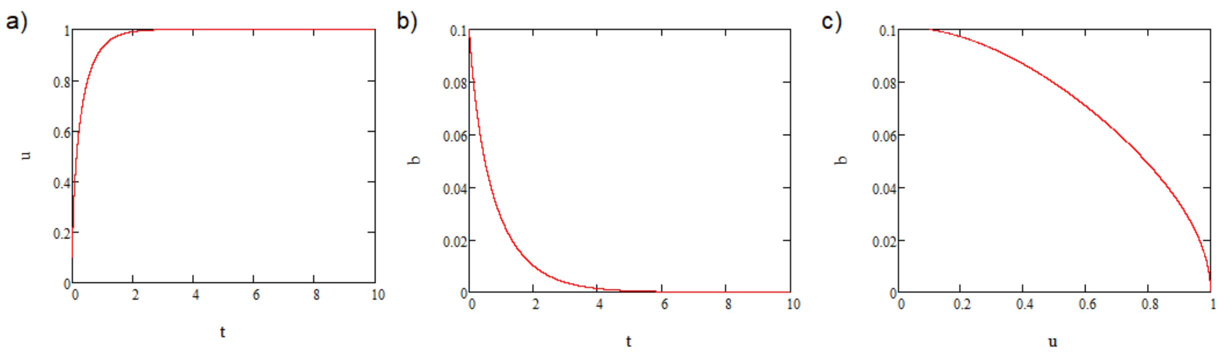

Fig. 3.

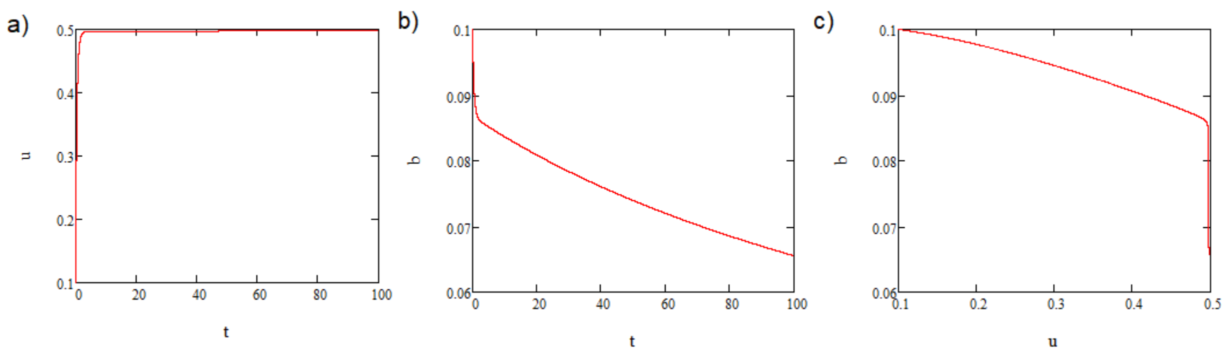

Fig. 4.

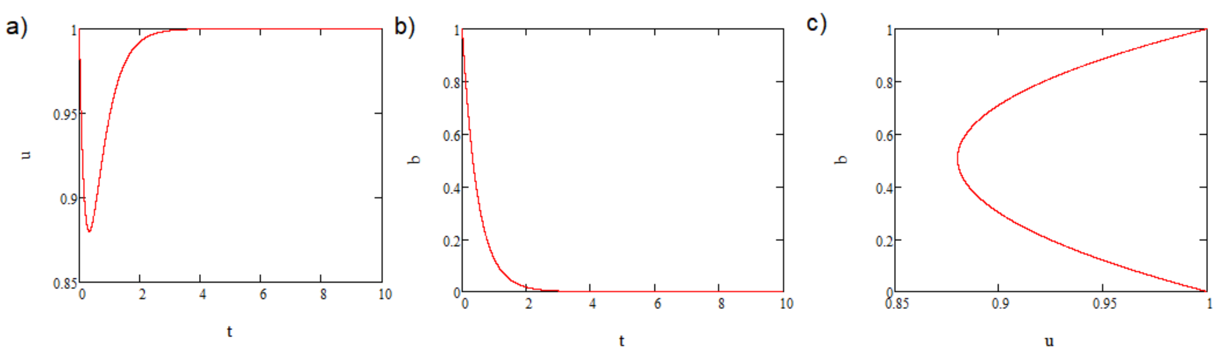

Fig. 5.

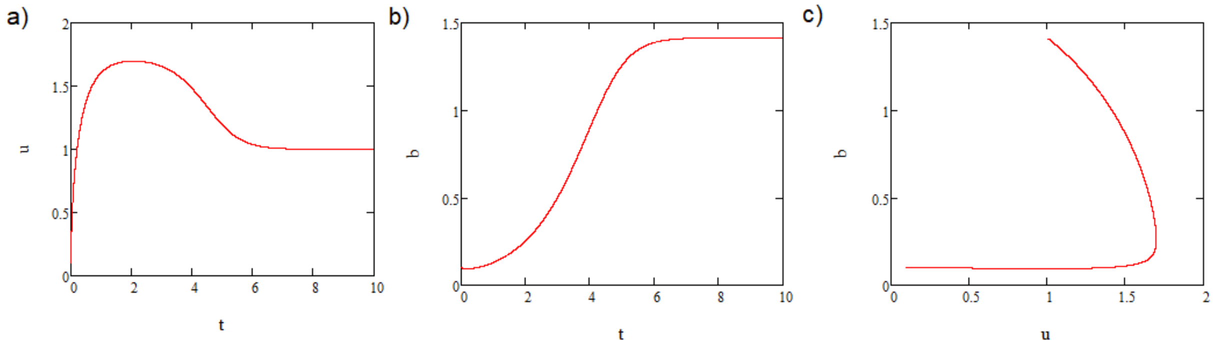

Fig. 6.